**Normal Distribution Exercises**

1. For a normal probability distribution N(0,1) find the 1, 2 and 3 quartile (0.25, 0.5 and 0.75), and the 0.95, 0.957, 0.99 and 0.999 percentiles.

**Answer**:

|  |  |
| --- | --- |
| Quantile | z-score |
| 0.25 | -0.674 |
| 0.50 | 0 |
| 0.75 | 0.674 |
| 0.95 | 1.645 |
| 0.975 | 1.96 |
| 0.99 | 2.327 |
| 0.999 | 3.091 |

1. Use the following information to answer the next two exercises: The patient recovery time from a particular surgical procedure is normally distributed with a mean of 5.3 days and a standard deviation of 2.1 days.
   1. What is the median recovery time?

**Answer**: 5.3

* 1. What is the z-score for a patient who takes ten days to recover?

**Answer**: 2.238

1. The heights of the 430 National Basketball Association players were listed on team rosters at the start of the 2005–2006 season. The heights of basketball players have an approximate normal distribution with mean, µ = 79 inches and a standard deviation, σ = 3.89 inches. For each of the following heights, calculate the z-score and interpret it using complete sentences.
   1. 77 inches

**Answer**: Use the z-score formula. z=–0.5141. The height of 77 inches is 0.5141 standard deviations below the mean. An NBA player whose height is 77 inches is shorter than average.

* 1. 85 inches

**Answer**: Use the z-score formula. z =1.5424. The height 85 inches is 1.5424 standard deviations above the mean. An NBA player whose height is 85 inches is taller than average.

* 1. If an NBA player reported his height had a z-score of 3.5, would you believe him? Explain your answer.

**Answer**: =3.5 deviations from the mean is very unlikely. Height =79+3.5(3.89)=92.62 inches, which is over 77 feet tall. There are very few NBA players this tall so the answer is no, not likely.

1. Kyle’s doctor told him that the z-score for his systolic blood pressure is 1.75. Which of the following is the best interpretation of this standardized score? The systolic blood pressure (given in millimeters) of males has an approximately normal distribution with mean μ=125 and standard deviation σ=14. If X= a systolic blood pressure score then X∼N(125,14).
   1. Which answer(s) is/are correct?
      1. Kyle’s systolic blood pressure is 175.
      2. Kyle’s systolic blood pressure is 1.75 times the average blood pressure of men his age.
      3. Kyle’s systolic blood pressure is 1.75 above the average systolic blood pressure of men his age.
      4. Kyles’s systolic blood pressure is 1.75 standard deviations above the average systolic blood pressure for men.

**Answer**: iv

* 1. Calculate Kyle’s blood pressure.

**Answer**: Kyle’s blood pressure is equal to 125+(1.75)(14)=149.5.

1. Some normal distribution has a mean of 34 and a standard deviation of 4. What is the probability of a random data point x falling between 21 and 45?

**Answer**: 0.997-0.001=0.996

1. In a test we got a scored 85. If the test scores were normally distributed, with a mean of 78 and a standard deviation of 5, what proportion of the class would likely score better than we did?

**Answer**: 0.080757

1. En una determinada región se estima que la temperatura máxima en el mes de abril sigue una distribución normal, con media 24° y desviación típica 6°. Calcula el número de días del mes en los que se espera alcanzar temperaturas máximas entre 22° y 28°.

Respuesta:

p(22 =< x =< 28) = p(Z1 =< z =< Z2) = p(z =< Z2) - p(z =< Z1)

Tipificando

Z1 = (22-24)/6 = -1/3; p(z =< Z1) = 0.3694

Z2 = (28-24)/6 =  2/3; p(z =< Z2) = 0.7475

p(22 =< x =< 28) = p(z =< Z2) - p(z =< Z1) = 0.7475 - 0.3694 = 0.3781

Si el mes de abril tiene 30 días, entonces

Cantidad de días con Temp entre 22 y 28 oC = 30 \* 0.3781 = 11.342 días

2. En el hospital Puerta de Hierro de Majadahonda suelen realizarse unas 10 operaciones de implantación de prótesis de cadera al mes. Estas operaciones son complejas y resultan en complicaciones en el 20 % de las veces. El problema es que el material protésico es costoso y caduca con facilidad. Por ello, el personal del hospital necesita saber cuántas prótesis deberían tener en stock para asegurarse que cubrirán las operaciones en un 95 % de los casos. ¿Sabrías darles la respuesta?

Respuesta:

Considerando que se tiene una distribución binomial en donde p(éxito) = 1- p(fracaso) = 0.8

Entonces  X ⟶ Bi(n,p); donde n = 10, p = 0.8

Aproximando a la normal X ⟶ ≃ N(n.p, sqrt(n ⋅ p ⋅ q) ); N(8,1.265)

p(x =< X) = 0.95 = p(z =< Z); Z = 1.6449

Dado que  Z = (X-μ )/σ ;  X = Z \* σ  + μ = 1.6449 \* 1.265 + 8 = 10.08

**Central Limit Theorem Exercises**

1. Suppose the population mean for the SAT score is 550 with a standard deviation of 75. Find the mean and standard deviation of the sample means for the following samples of sizes:

a. n =50

**Answer**:

The mean of the sample means is 550. The standard deviation of the sample means is .

b. n = 200

**Answer**:

The mean of the sample means is 550. The standard deviation of the sample means is .

c. n = 800

**Answer**:

The mean of the sample means is 550. The standard deviation of the sample means is .

d. n =3,200

**Answer**:

The mean of the sample means is 550. The standard deviation of the sample means is .

1. The average GPA scored by a class is 4.91 and standard deviation is 0.72. For a sample of 20 students, find the z-score that the average is above 5.

**Answer**:

1. The average score of a subject is 2.89 for the whole class, with a standard deviation of 0.63. If a sample of 25 students is being taken, then find the probability of getting the average of this sample to be more than 3.

**Answer**:

Hence, the probability of the average being larger than 3 is 0.191

1. In a survey of a company, mean salary of employees is 29,321 dollars with SD of 2,120 dollars. Consider the sample of 100 employees and find the probability their mean salary will be less than 29,000 dollars?

**Answer**:

Hence the probability of employees having mean salary less than 29,000 dollars is 6.6%.

1. A large freight elevator can transport a maximum of 9800 pounds. Suppose a load of cargo containing 49 boxes must be transported via the elevator. Experience has shown that the weight of boxes of this type of cargo follows a distribution with mean μ = 205 pounds and standard deviation σ = 15 pounds. Based on this information, what is the probability that all 49 boxes can be safely loaded onto the freight elevator and transported?

**Answer**:

We are given n = 49, μ = 205 ,σ = 15. The elevator can transport up to 9800 pounds. Therefore these 49 boxes will be safely transported if they weigh in total less than 9800 pounds. The probability that the total weight of these 49 boxes is less than 9800 pounds

is

1. The amount of regular unleaded gasoline purchased every week at a gas station near UCLA follows the normal distribution with mean 50,000 gallons and standard deviation 10,000 gallons. The starting supply of gasoline is 74,000 gallons stored in the tank, and there is a scheduled weekly delivery of 47,000 gallons.
   1. Find the probability that, after 11 weeks, the supply of gasoline will be below 20,000 gallons.

**Answer**:

The total amount of available gasoline is T=47,00x11+74,000=591,000. The supply will be below 20,000 gallons if the total gasoline purchased in these 11 weeks is more than 591,000 – 20,000 = 571,000 gallons. Therefore we need to find

* 1. How much should the weekly delivery be so that after 11 weeks the probability that the supply is below 20,000 gallons is only 0.5%?

**Answer**:

Let A be the unknown schedule delivery. Now the total gasoline purchased must be more than 74000 + 11×A−20000. We want this with probability 0.5%, or

. The z value that corresponds to this probability is 2.575. So, . The weekly delivery must be 52,854.8 gallons.